Periodiek

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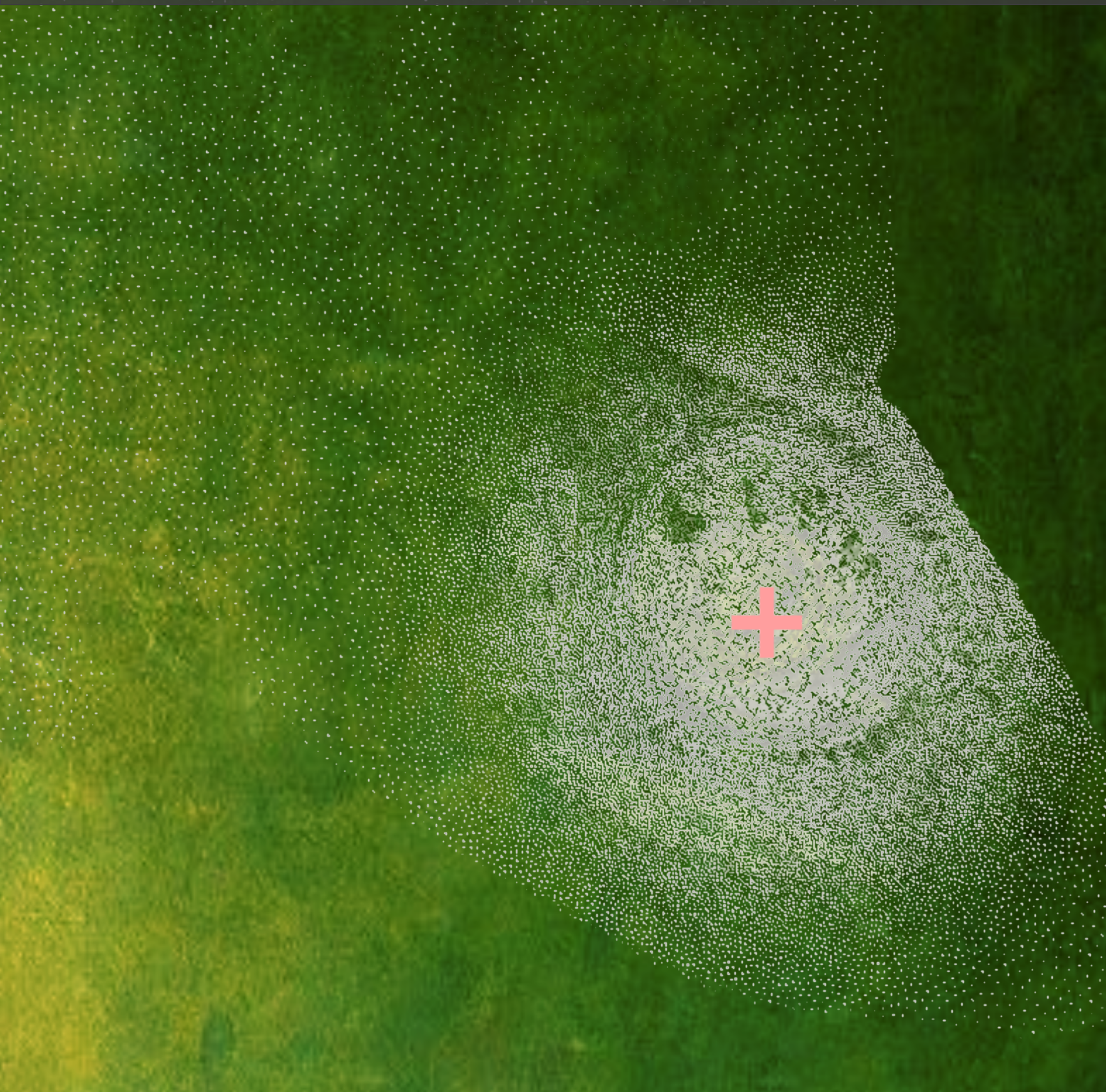
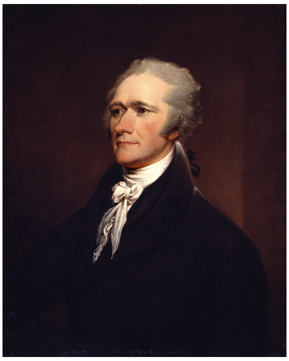


Perio Interview: Steven Hoekstra

Well known among older students from previously teaching Electricity & Magnetism, Steven Hoekstra is an experimental physicist, whose main interests are cold molecules and precise measurement systems. If you want to learn more about his research and personal life, make sure to have a look!

Hamilton vs. the Alabama Paradox

"Winning is easy, governing is harder." Now that we have your attention, how about some math? In this article, we will examine one of the common ways of assigning electoral seats: the Hamilton Method. Using the already established criteria for fairness, we will explore the method's shortcomings with different examples.



Quiz: Which board member are you?

Now that half of our not-so-new board's year has passed, we expect the FMF-ers to already know the people in charge of our favourite study association pretty well. However, few of you have probably given much thought to the question which one of them is the most relatable. Worry not! You have the chance to find out now.

In this Periodiek

From the Editor in Chief

New room, new Perio*diek*. While we get used to our temporary room, we can all lean back and remember some of our fond memories of room 5111.0053. Have a look at page 23 for a micro-dose of nostalgia.

This issue we're happy to bring you a double dose of board news. Read about Rolf and Roy on pages 4 and 6 before finding out who you are more like by taking the quiz on page 14.

On the academic side, we have some interesting research by the Bernoulli Institute about Foveated Rendering (page 18) which got featured on the front page.

Steven Hoekstra also makes an appearance this issue to share a bit about his career and personal life, check out page 10 for the full interview.

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From the Board
Treasurer

AUTHOR: **R. DEKKER**

I am Rolf, the treasurer of the sixty-fifth board of the FMF. Me and Roy had originally planned to get together, have a couple of beers, and write something for the Periodiek together. Alas, we kept somehow being too hungover, or drunk, to actually accomplish this. Now we've approached our deadline and decided to start writing our pieces alone instead.

Of course, I am not sober whilst writing this piece and I don't expect Roy to be either. Luckily this is what the members have come to expect from us, so it fits right to our characters.

Naturally, you all expect a little update on what the board has been doing in this section, especially since it is called "From the Board". Since the last edition of the Periodiek, I have settled into my role as treasurer a lot more. However, I've also been finding out how much I still need to learn. Someone get me some Ketel.

One of the things that I've been hard at work for is the return of Monthly incassos. Surprisingly I've been getting mostly positive responses to charging people money monthly.

"I am not sober whilst writing this piece and I don't expect Roy to be either."

It is quite shocking to think I'm already almost halfway through my board year. It has been such a wild, but awesome experience. Even though I wasn't initially planning to be doing this at all this year I am so glad I am. I must be biased, however, because I have a board of people that I absolutely adore and wouldn't change for the world.

They frustrate the hell out of me, as I am sure I do them, and yet we always manage to end up having a great time together.

With the half-year GMA rapidly approaching we can take a quick glimpse towards the second half of the year. I'm sure you're all happy to hear that we're running out of dates for events because our amazing committees have been hard at work to plan lots of exciting activities. And even though as board we're taking a slightly different approach to May month, it is still looking to be the highlight of the year.

"Someone get me some Ketel."

I fear I have written far too much already and therefore will finish off my piece here. There are still many more things for you all to read in this edition of the Periodiek. If you see me or Roy around, come have a beer with us and make it a great second half of the year.

Figure 1: Rolf at Kloktober

Commissioner of Educational Affairs

AUTHOR: **R. RODENBURG**

Hello, I am Roy, and this year, I am the commissioner of educational affairs at the FMF! Time has been passing fast, especially now that I have almost become a permanent resident of the FMF room.

Contrary to what Rolf suggests, I am writing this sober since I have slightly overshot the submission deadline *(sorry Perio)*. Therefore, I am now writing it during the day. Hopefully, that will help make my piece a bit more coherent.

I can barely comprehend how much of this year has already passed. We have now passed two-thirds of the academic year, and only now do I see myself getting to grips with the finer points of my work.

In contrast to my fellow board members, I am getting towards the end of my time as a full-time student. I started studying in 2019 for a double bachelor's degree in mathematics and physics. I trudged through my bachelor's during Corona by dedicating my life to my courses. I finished my bachelor's in 2022 with a strong desire to get a taste of life beyond courses. So when I started my master's, I joined the FMF. I quickly felt at home, and I joined multiple committees which I really enjoyed doing.

"Contrary to what Rolf suggests, I am writing this sober since I have slightly overshot the submission deadline."

I helped organize the Mathematics symposium and borrel lectures, which slightly foreshadowed my future role on the board, although I didn't notice myself. I decided about halfway through the year to apply for the board at FMF since I would only have a few courses left to take before starting my master's thesis, making this year the natural moment to take time off from courses.

This year I have been trying to take the most out of student life, which a board year definitely allows for if you take few courses and have the ability to stomach your hangover and still open the room at 9:00 and attend that meeting someone so foolishly put at 9 am. As a board member, you get invited to a lot of events and parties for which you usually wouldn't get invited to.

For example, we got invited to the opening of Belsimpel's new office at de Grote Markt, a nice event that was significantly less grimy and sticky than the usual student fare. Besides those kinds of events, my favourite moments of this board year have been the unexpected ones: days when you expect to be home at 6 pm, but find yourself in bed at 6 am.

Figure 2: Roy holding a beer

Hamilton vs. the Alabama Paradox
Why no house of representatives is fair?

AUTHOR: **R. MOL**

Imagine you are in charge of a nation. You have drawn up borders between your states and set up a house of representatives. You decide that each state should be represented proportional to the population of that state. A state with 10% of the total population should have 10% of seats in the house of representatives. While that sounds reasonable, you have already set yourself up for failure.

Introduction

For example, say you have a population of 10000 people, 10 seats, and 2 states. This means each seat represents 1000 people. Let's say one state has a population of 4200 people and the other 5800 people. Because you cannot share a seat, the first state will receive 4 seats and the second will receive 6. Here, we see that states almost always will have to be rounded up or down. However, we can't always simply round each according to conventional mathematical rules. Consider for this the same example but with 3 states, with the population of the states being 3750, 2650, and 3600 respectively. Conventionally, we would round all of them up, but then we would end up with 11 seats and we only have 10. So one state will have to get rounded the 'wrong' way. This is a fact of life that we have to accept.

The method we just described is known as the 'Hamilton's Method', named after Alexander Hamilton[1], and assigns seats as follows: First, determine the number of seats each state gets as a decimal number. Then, assign each state its whole number part and rank the decimal part in ascending order. Count the number of seats not yet assigned and hand them to the states highest to be rounded up.

The founding fathers of America tried to come up with several ways to tackle the issue of how to assign seats, and all of them fall short in at least one criterion that is generally accepted as necessary for the system to be considered 'fair'. In this article, we will learn about those shortcomings in the context of the Hamilton Method. We will not discuss why these criteria cannot co-exist, but instead show some examples of when they show up.

Criteria for fairness

While 'fairness' is a highly subjective subject, the criteria below are hopefully so fundamental that everybody can agree that their presence is required. More specifically, I hope we can all agree that not having any of these criteria would result in a flawed system.

Criterion 1: The Quota Rule.

When rounding the amount of seats of a given state, we have to change a decimal number into a whole number. It seems fair to either round it up to the integer above it, also known as the upper quota, or the integer below, also known as the lower quota. For example, if you have to round 6.2, you would either round it to 6 or to 7. This criterion is called the quota rule.

This seems like a straightforward rule. We've seen in an earlier example that some states will have to be rounded the wrong way but we would always round every state to either the upper quota or the lower quota. As we will see, however, this rule might be the only one that stands in the way of a completely fair system.

Criterion 2: House Monocity, the Alabama Paradox.

Suppose that seats are assigned to our states and have been rounded. Now suppose we decide to increase the amount of seats in our house of representatives for some reason (perhaps an increase in population or a remodeling of the government building that allows for more chairs, so more seats). Since the amount of seats a state gets is decided by their portion of the population multiplied by the total amount of available seats, and because now have more seats to hand out, we expect some states to gain a seat, some to keep their amount of seats, but no state to lose any seats.

[1] For more information, see "Hamilton: An American Musical"

$$\text{Amount of seats in state } x = \frac{\text{Population of state } x}{\text{Population of the nation}} \times \text{Amount of seats in the House of Representatives}$$

As you might have guessed, this is not always the case: consider a third example of 6 states, 20 seats, and a population (and seat) division represented in the table below:

State	Amount of seats before rounding	Whole seats	Rounding order	Amount of seats after rounding
A	5.2	5	5th	5
B	1.4	1	2nd	2
C	2.6	2	1st	3
D	4.15	4	6th	4
E	4.3	4	4th	4
F	2.35	2	3rd	2
total	20	18	-	20

Here, the rounding order is the order in which we round seats up, we first round the state closest to a higher integer, then the next, and so on. So here state D is the last state to get rounded since 0.15 is lower than the decimal part of every other state. We assign each state at least their whole number part and then assign extra seats based on the rounding order. Since we have 2 extra seats to assign, they will go to numbers 1 and 2, states C and B.

This system seemed all good and fair until Alabama lost a seat after an increase of seats in the house. If we take the above example and add 2 seats to the house, increasing the amount by 10%. Then, we get the following table:

State	Amount of seats before rounding	Whole seats	Rounding order	Amount of seats after rounding
A	5.72	5	3rd	6
B	1.51	1	6th	1
C	2.87	2	1st	3
D	4.565	4	5th	4
E	4.73	4	2nd	5
F	2.585	2	4th	2
total	22	18	-	22

If we compare this to the first table, we can immediately notice two things. Namely that the rounding order has changed and that we have 4 seats to assign instead of 2. This means that now the first 4 states get rounded up instead of the first 2. But since the order has changed, the original first 2 seats don't have to be in the new highest 4. Indeed, state B gets rounded down although it used to be rounded up. This is known as the Alabama Paradox, and we would like our system to avoid this paradox.

It is no coincidence that state B is the smallest state. Because all states get 10% more seats, larger states increase their share more (in absolute terms) and this means they are likely to overtake smaller states in the rounding order.

This is an example of House Monocity: if the amount of seats increases, no state should lose seats. A different way this can be violated is if a nation annexes a nearby region and adds it as a new state. This would increase the population and even if we increase the amount of seats in such a way that each seat still represents the same number of people, the ordering might still change (Inigo et al., 2021).

Criterion 3: Population Monocity.

Of course, the population of any state is dynamic. Every year, people are born and people die. Similarly to House Monocity, if the population of one state increases more than that of another state, the faster-growing state should not lose a seat to the slower-growing state. In the next example, we will see how this can happen using Hamilton's method. The attentive and skeptical reader will point out that we can measure growth in more than one way, in absolute numbers or as a percentage. In the example below, the state we call attention to grows faster in both regards.

Consider the following example with 3 states, 30 seats, and a population division that is as follows:

State	Population	Amount of seats before rounding	Amount of seats after rounding	Population increase	Population increase (percentage)
A	55	1.65	1	2	3.64%
B	256	7.68	8	18	7.03%
C	689	20.67	21	80	11.61%
total	1000	30	30	100	-

Recalculating the number of seats using Hamilton's method after the population increase gives the following result.

State	New population	New amount of seats before rounding	New amount of seats after rounding	Change in seats
A	57	1.55	2	1
B	274	7.47	7	-1
C	769	20.97	21	0
total	1100	30	30	-

We can see that state C grew the fastest, followed by state B, while state A grew the least. Here, this order is the same in absolute numbers and percentually. This is purely incidental (or rather, by design) to keep the example digestible. If the orders were different, it would make more sense to look at the percentual change, rather than the absolute change, since seats are also assigned proportionally, i.e. percentually. We also only consider the case in which the population of each state is growing.

We could also have an example of a rapidly shrinking state losing fewer seats than a slowly shrinking state but the example would not show us much new and only clutter up the message. As we can see here, state B lost a seat to state A, even though the population of state B grew more.

Discussion

Having seen these 3 criteria, it might feel like this method is very prone to unfairness. These examples are deliberately chosen to reflect the flaws of Hamilton's method. Of course, we can think of examples where the method runs into no problems at all, but we want a method that works all the time, for all divisions of the population. And such a method, unfortunately, cannot exist. In this text we focused on Hamilton's method, which passes the Quota Rule but does not pass the House Monocity or Population Monocity criteria. Other methods have been thought of to counter Hamilton's method, such as Adam's Method and Jefferson's Method, but they run into their own fair share of problems. The methods revolve around gradually changing how many people each seat represents until the right amount of seats is reached. While this solves a lot of problems, it creates new ones, where the method now passes the House Monocity and Population Monocity criteria but not the Quota Rule (Inigo et al., 2021; Parker, 2021).

The conclusion that no method can fulfil all three criteria comes from a work titled "Fair representation" written by Balinski, Young, and Peyton in 1982. Filling nearly 200 pages, it is not conceivable to explain in detail here. The take-away message we are interested in is that no method can satisfy all three criteria. So there are no 'fair' methods if we deem all three criteria as necessary.

Closing remarks

Something not mentioned thus far, but related to the issue we have portrayed, rounding states changes the amount of people their seats represent. If we go back to a nation with two states, one with 3300 people and the other with 5700 people and a total of 9 seats, then giving the first state 3 seats and the second state 6 seats results in the first state having 3300 people per 3 seats, or 1100 people per seat, while the second state has 5700 people per 6 seats, or 950 people per seat. This creates a different type of unfairness which is separate from which method we choose, although we cannot rule out that some suffer more from this issue than others. This problem, of having some people being represented relatively more than others, becomes smaller as we increase the number of seats, since we will have to round relatively less, e.g. rounding 1.6 to 1 or 2 is a bigger change than rounding 12.3 to 12 or 13. By ratio that is, one is a decrease of 37.5% or 25% for rounding down and up respectively, and the other is a difference of 2.5% or 5% for rounding down or up respectively. Of course, if we make everybody a politician in the house of representatives, we would not have to round, but can you imagine the world we'd live in?

On a technical note, the population here might seem unreasonably small. If this bothers you, consider it instead to reflect the population of a state not in single people but in 10.000 people. Similarly, some examples here only have 2 or 3 states. While this isn't entirely out of the realm of possibilities, it helps to think of them as a few states among tens of states in a nation, but with the other states not having any interesting numbers to draw conclusions from.

There are of course other factors that have been deciding in which methods can and can't be used. Such as, for the US, ones dictated by the constitution which don't have much to do with the problem described above. While those restraints would certainly give a more interesting problem of selecting a method of dividing seats, it would also complicate matters greatly, too much in order to be a nicely digestible read.

This topic is far wider and more interesting than what was able to be captured on a few pages. Some of the sources below are great for learning more about this topic in varying levels of complexity. In particular, the video by Parker is a great entry into the subject, whereas the work by Inigo et al. provides a few more examples in section 9.3.

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Perio Interview: Steven Hoekstra
Researcher, Teacher, Musician

AUTHORS: **I. BALINT, S. OLTEANU, Y. SAVOVA**

Steven Hoekstra, known for teaching Electricity and Magnetism, currently teaching Waves and Optics, is an experimental physicist at the University of Groningen. In addition to trapping cold molecules, you can find him playing the clarinet in his free time. In this interview, we will get to know him better.

What motivated you to become a professor?
That's a difficult question because that's something that gradually happens. So of course, as a kid, I was curious by nature, I wanted to find out how things work and learn more about the mechanisms behind what we see. That's of course a very general statement: I liked Physics so I started studying. However, I realized that if I have to do things I don't really like, I'm not very good at them. There's a very strong connection between what I like and how good I am at it. Doing a PhD seemed a good natural follow-up, then I thought that perhaps I could try to do a postdoc and found this position at the University and kind of gradually continued like that.

Figure 3: Steven Hoekstra

So you decided to study Physics because you liked it from a very young age?
Well, not very young, I think I was at the end of high school. I was 17 or so and I was thinking: "Oh, what should I do?". There were two main options: Architecture and Mathematics. A friend of mine was already doing Mathematics, so I thought, OK, he already picked Mathematics, then I should do something else, and I chose Physics because I also liked it, but it was not like Physics, Physics, Physics, Physics, all the time. I think when you start studying something, you don't really know what it's going to be like. I just chose based on the image I had and it worked out well, it was not completely different than what I expected.

So you never regretted choosing Physics?
No, I don't regret it. I think it was a good choice, but I could also have had lots of fun and perhaps success in a different field. I don't think I was just born to do Physics and that's it. I guess if my friend had picked Physics, perhaps I would have done Mathematics, or Architecture, or something else.

Can you tell us more about the field of Physics that you're most interested in?
I work on atoms, molecules, quantum physics, and lasers. Experimental work. We try to do very precise measurements by trapping a molecule or an atom for a long time. I find it fascinating that you can just completely isolate these particles. You have all these pretty complicated transitions at first, but it's rather simple in the sense that there's a handful of ingredients like electrons and protons, which build a nice system that you can almost completely understand. So you can study very detailed interactions between particles, and that can tell you something about matter and antimatter asymmetry that is so relevant on a universal scale. At the same time, what I like most is just to go into the lab and build something and work with the students.

So you're more interested in experimental work than theoretical work?
Yeah, I think so. It's not that I don't like theoretical work. I like to do some simulations, but I'm not very good at analytical, complicated derivations. I kind of start to lose interest pretty quickly. Perhaps I'm a bit too impatient for that.

So you mentioned a few other options that you were considering. If you didn't study Physics, what would it have been?
I think it would have been something like natural sciences, so it could also have been Biology, but I don't think Psychology, Law or Business. I also would have liked something creative. Like I said, I was considering Architecture, because it involves technical aspects with creative work: the building should not collapse after all and it should be a nice place to live in as well. Perhaps a combination like that would've also been nice.

Can you tell us more about where you're from originally and how you ended up here?
Yeah, that's easy. I'm from Groningen. I was born in the city and pretty soon my parents moved to Haren on the south side of the city. My father was working for the University as a biologist. So I went to primary school here in Groningen and then I started studying Physics at the University. Then I went to Berkeley, California to do research for a year. After that, I wanted to do a PhD and I had two options: Amsterdam and Groningen, and I chose Groningen. So I did my PhD here and then I moved for five years to do a postdoc on cooling molecules in Berlin. There was a tenure track position in Groningen, which I got in 2010. So that's how I ended up here in Groningen: always trying to get away, but also trying to come back. Now I accept that Groningen is my destiny, at least so far.

So because you've lived here for a very long time, what is your favorite thing to do in the city, or place to go to?
I like the Vismarkt a lot, the market on Saturday. I like going to concerts, so of course Oosterport and Schouwburg. I like eating out in the city center. Of course, it's great that Groningen has a lively city center, but outside the center, where I live, I enjoy nature. You're immediately in the fields with the cows and the wind and the sun. Yeah, I like that.

So what research are you currently working on?
I'm working on two projects. One is called electron EDM, so the electric dipole moment of the electron, a big project that I lead, with many people involved. The other one that we started five years ago is levitated nanospheres, which are small spheres of about 150 nanometers that we put in the focus of a laser beam on an optical tweezers. They oscillate in this light focus, forming a system that can be used as a very sensitive sensor for accelerations and forces. You can also study the transition from quantum objects into classical objects. It's kind of a smaller experiment with a smaller team, but it's a very nice project as well.

"Now I accept that Groningen is my destiny, at least so far."

Can you rank the courses that you teach or tell us your favorite?
Well, it's difficult to rank, right? I taught Electricity and Magnetism for a long time. Now I stopped doing that, which is a pity, but it's also good that somebody else will do it and I moved on to other courses. For Waves and Optics, I like it a lot because the subject is really nice. Electricity and Magnetism is also really nice but there's a need to follow the book closely to solve all the exercises; it's lots of new concepts, so that's nice to guide. For Waves and Optics, I have a bit more freedom as a teacher to make the course different. I tried to change it quite a bit from how it was before and I think it's nice to give more responsibility to the students to make their own projects instead of doing cookbook recipe standardized things. I started teaching The Quantum World last year, which is in the minor, so it's quantum physics for non-physics students. That's also fun but completely different because it's without mathematics which is a nice challenge. So at the moment, I don't really have a favorite course, they're all a bit different. I enjoy teaching a lot.

What is your favorite equation?
I don't know whether I have a favorite equation since I'm not so much driven by equations. I mean of course, from teaching Electricity and Magnetism, I like Maxwell's equations. They're very compact in an elegant way. For quantum physics, you could say the Schrödinger equation, showing the wave function and everything that you can do with it. So I would say those are nice examples, but unfortunately I don't have a T-shirt with a favorite equation.

Do you have any colleagues that you admire and you like working with?
Yeah, quite a few of my colleagues. I think that it's nice that for this electron electric dipole moment project that has been running for multiple years, we work with a team of up to 20 people. It needs a team because there's particle physics theory, quantum chemistry, and experimental physics. There are many components that we need to do together because I cannot do it alone. So, for example, my colleague Anastasia Borschevsky works with quantum chemistry calculations and she can really work with her team. She can also write papers really fast, which is nice of her. But on the experimental side, there's my colleague Lorenz Willman, who is just very good at keeping things working in the lab, setting it up in a reliable way, and setting the data infrastructure. Then there's the particle physics theory colleagues, like my colleague Rob Timmermans, who is involved in the particle physics aspect of it and he looks at the implications of a measurement of a molecule for the fundamentals of particle physics, and that's something that he has a very good overview of that I don't have myself.

So these were our academic questions, now we would like to move on to more lighthearted questions. So, what do you like to do in your free time?
I go running once or two times a week, if I find the time. I have two sons, and they do sports, survival run. Perhaps you know, there's this trek out there. Every Saturday morning, they do that for one and a half hours and in the meantime I go running, with a couple of friends or alone. I also like photography, so I'm part of a photography club where perhaps once a month we get together and do a workshop on a topic. I also make music. I like music quite a lot. I play clarinet in a woodwind ensemble, which is again totally different. We rehearse about once a month and then once or twice a year we give a concert. And I like cooking and food in general. So yeah, last week a PhD student of mine graduated and gave me a cookbook of Persian dishes with some ingredients. This weekend I made my first dish from that cookbook. So that was fun also.

So how did you get into playing the clarinet?
I started playing when I was about 10 or perhaps a little bit younger. I tried different instruments and I picked the clarinet and of course for some time it was difficult to practice every day if you don't really like it. Practicing is an important part of it, because otherwise you don't get better. At some time as a kid, perhaps you're not intrinsically driven so much, so you need your parents to guide you through that a bit to keep practicing. Then as a student, I started playing again quite a lot because I got accepted for the student orchestra, so I played in Groningen. And I auditioned and got selected for the National Student Orchestra, which was great: you do a month-long project where two weeks you rehearse and two weeks you tour all the concert halls in the Netherlands. We also went to Ireland to give a concert there. So that was really great. And I met my wife in the orchestra, which is also important. So in that sense music is quite a big part of my life.

"You're immediately in the fields with the cows and the wind and the sun. Yeah, I like that."

What kind of music do you usually listen to?
Ohh, that's a mix of everything. I like baroque music, really old stuff like Monteverdi, for example. During Corona times I tried to learn to play a different instrument, it's called zink or cornett. It's like a wooden trumpet in a way. I'm not very good at it, but I later learned that it's one of the most difficult instruments to play because there's a tiny mouthpiece and you really need to have lots of pressure in your lips to be able to do it. I also listened to lots of more modern music. In high school I was a fan of Jimi Hendrix, Captain Beefheart and Frank Zappa, so kind of more, let's say, alternative music from that time. But now I also try to follow what my kids listen to and we also try to make music together. So for example, my oldest son plays bass guitar and also cello, but he likes bass guitar more I think, and so we recently got also a drum kit. So now we kind of have a small band at home because my youngest son is playing piano and my wife plays cello and sings. So we can kind of make music together.

What do you usually like to cook?
I like cooking Italian food because it's always delicious and the kids are also happy. Yeah, like I mentioned, I tried this Persian dish with saffron, which is a different style. Also at home we have tried in recent years quite a bit of Japanese and Korean food. We have a nice Japanese cookbook with vegan recipes. We eat vegetarian at home because my youngest son is vegetarian. So most days we don't eat meat or fish.

Do you find time to cook every day?
No, but my wife also cooks actually, and she cooks a bit more than I do. I cook two or three days a week and she cooks the rest. But also now my oldest son cooks regularly. So that's also nice. We do eat something that we prepare ourselves, I think almost every day. Sometimes, of course, we get a pizza.

In a zombie apocalypse type of scenario, which other professor would you want to have on your side in order to survive?
That's an original question. Zombie apocalypse? So what kind of skill set would be important then? I guess we're threatened and we need to survive. Yeah, it's either fighting or hiding and gathering resources. Who would be good at that? Or running? So I have some colleagues that I run with sometimes, like Caspar van der Wal, so I know he can run and it would be one option just to run away together. I know Anastasia is also good with collecting food, so she knows mushrooms and those kinds of things, that might also be a good skill to have. So that at least if you cannot get to the store, you can get your own food. And regarding fighting or at least defending, that perhaps I would try to avoid.

What are you most looking forward to in the future?
I actually look forward to moving to our new building because the current one is a bit isolated from the rest of the campus. It would be nice to be more integrated with the rest and the labs will also look very nice once they are complete. So that's something I look forward to. And I have a couple of trips planned in the coming months, to Germany, Italy, and the US. So it is always nice to break a little bit of the pattern and just travel, talk to other people, and experience a bit of a different culture. And food.

Is there anything else that you would like to mention?
No, I don't think specifically. Perhaps indeed many of the students at some point need to pick a project to work on for the Bachelor or Master thesis. This year it is difficult because many of the experimental groups are moving, but usually I really like supervising those kinds of projects and quite a few students find their way to us, which is nice. I look forward to being able to do that again full power, hopefully next year, after the move.

Rapid fire questions

What is your favorite food?
Italian.

Favourite color?
Green.

Favorite season?
Spring.

Favorite book?
The 1000 autumns of Jacob de Zoet by David Mitchell.

Favorite movie?
Again many different possible options. I like these animated Japanese movies by Hayao Miyazaki.

What languages do you speak?
Dutch. I can understand Frisian. English. I can do reasonably well in French. I can understand a bit of Italian. And German of course, because I lived in Germany for five years. I'm learning Japanese now. I have a 200 day streak of Japanese on Duolingo.

Do you prefer cats or dogs?
Dogs. Yeah, we have a dog at home, so usually I start my day with a half an hour walk with the dog. I don't dislike cats, but my dog does, but he likes the chickens. We have 3 chickens and we get their eggs and we leave the chicken alive. But that's also a nice animal, you can add it to the question.

"I have a 200 day streak of Japanese on Duolingo."

What did you have for breakfast?
I had muesli with yogurt for breakfast. I alternate between bread and muesli for breakfast. And coffee and tea.

Which board member are you?
Take the quiz!

AUTHOR: **R. MOL**

Have you ever wondered which board member could be your soulmate? Find out what you have in common with the board members of the FMF and get to know them a little better by taking the quiz and maybe find your new bestie.

What is the best soft drink?

Cassis
Dr. Pepper
Grape Fanta
Orange Juice
Sprite

What is the worst soft drink?

Cassis
Fristy
Orange Fanta
Tomato Juice
Tonic

What is your favourite animal?

Chicken
Gecko
Human
Polar bear
Sugar Glider

What is your favourite beer?

Cherry Chouffe
Hertog Jan
"I can't tell the difference"
Weihenstephaner
Weizener

Pick a superpower

Flying
Mind reading
Not being hungover
Teleportation
Waterbending

How long does it take you to chug an Ice?

5.79 seconds
18.62 seconds
22.47 seconds
52.57 seconds
60+ seconds / not at all

What is your go-to snack at the FMF?

Cup-a-soup
Other people's tostis
Paprika chips
Toblerone
Torondos

What is your favourite musical?

Big Fish
Hamilton
"I don't do musicals"
Phantom of the Opera
Yellow Submarine, with Kiselev

An Ice is hidden in the FMF room, you..

..avoid it / pretend not to see it
..forget and find it anyway
..go look for it to chug it
..helped hide it
..would look for it to sip it quietly

What is your least favourite Disney movie?

"Any that I watched as an adult"
Frozen
The Last Jedi
Toy Story
Sleeping beauty

Pets?

A British Shorthair named “Solo”
Currently not, but want to
Don’t have any
Three cats and one (very stupid) dog
My mom has horses

What sport do (or did) you participate in?

Cycling
Field hockey & Running
Gym
Martial arts (Aikido & kickboxing)
Swimming

You have an exam in two weeks, you..

..do nothing, study two days beforehand, fail the exam
..go get a drink
..started studying two weeks ago
..try to start, end up studying four days in advance
..would stress out, and start studying

What weird talent/skill do you have?

Bending my pinky so far back it touches the back of my hand
I chug beers fast
Losing my keys
Making people lick the floor
Remembering song lyrics

What is your religious background?

Believed in God at age 11 so he would grant me favours (it did not work)
Jehovah’s Witness
My mom went to church weekly and hated it. So I avoided it.
Orthodox family, my grandma sent me here with a bottle of holy water
Very religious baptist

Answers!

Now that you completed the quiz, you have a chance to compare the answers with the board's.

What is the best soft drink?

Cassis	Roy
Dr. Pepper	Andra
Grape Fanta	Stefi
Orange Juice	Femke
Sprite	Rolf

What is the worst soft drink?

Cassis	Rolf
Fristy	Femke
Orange Fanta	Stefi
Tomato Juice	Andra
Tonic	Roy

What is your favourite animal?

Chicken	Femke
Gecko	Stefi
Human	Roy
Polar bear	Rolf
Sugar Glider	Andra

What is your favourite beer?

Cherry Chouffe	Andra
Hertog Jan	Roy
"I can't tell the difference"	Stefi
Weihenstephaner	Rolf
Weizener	Femke

Pick a superpower

Flying	Femke
Mind reading	Stefi
Not being hungover	Roy
Teleportation	Rolf
Waterbending	Andra

How long does it take you to chug an Ice?[2]

5.79 seconds	Rolf
18.62 seconds	Stefi
22.47 seconds	Roy
52.57 seconds	Femke
60+ seconds / not at all	Andra

What is your go-to snack at the FMF?

Cup-a-soup	Stefi
Other people's tostis	Femke
Paprika chips	Andra
Toblerone	Rolf
Torondos	Roy

What is your favourite musical?

Big Fish	Rolf
Hamilton	Andra
"I don't do musicals"	Femke
Phantom of the Opera	Stefi
Yellow Submarine, with Kiselev	Roy

An Ice is hidden in the FMF room, you..

..avoid it / pretend not to see it	Roy
..forget and find it anyway	Stefi
..go look for it to chug it	Rolf
..helped hide it	Femke
..would look for it to sip it quietly	Andra

What is your least favourite Disney movie?

"Any that I watched as an adult"	Roy
Frozen	Stefi
The Last Jedi	Rolf
Toy Story	Femke
Sleeping beauty	Andra

[2] Data provided by Escalacie.nl

Pets?

A British Shorthair named "Solo"	**Andra**
Currently not, but want to	**Roy**
Don't have any	**Rolf**
Three cats and one (very stupid) dog	**Stefi**
My mom has horses	**Femke**

What sport do (or did) you participate in?

Cycling	**Roy**
Field hockey & Running	**Femke**
Gym	**Rolf**
Martial arts (Aikido & kickboxing)	**Stefi**
Swimming	**Andra**

You have an exam in two weeks, you..

..do nothing, study two days beforehand, fail the exam	**Andra**
..go get a drink	**Rolf**
..started studying two weeks ago	**Roy**
..try to start, end up studying four days in advance	**Stefi**
..would stress out, and start studying	**Femke**

What weird talent/skill do you have?

Bending my pinky so far back it touches the back of my hand	**Stefi**
I chug beers fast	**Rolf**
Losing my keys	**Femke**
Making people lick the floor	**Roy**
Remembering song lyrics	**Andra**

What is your religious background?

Believed in God at age 11 so he would grant me favours (it did not work)	**Roy**
Jehovah's Witness	**Rolf**
My mom went to church weekly and hated it. So I avoided it.	**Femke**
Orthodox	**Stefi**
Very religious baptist	**Andra**

On Foveated Rendering
Research by the Bernoulli Institute

AUTHORS: **C. TURSUN, S. FREY, AND J. KOSINKA**

Foveated rendering is a technique in computer graphics and visual computing that leverages the biological characteristics of the human eye to optimize resource allocation during rendering. The human eye has the highest visual acuity in the central area of vision, known as the fovea, while peripheral vision is less capable in resolving visual details. By rendering high-resolution graphics where the viewer's gaze is focused (the foveal region) and progressively decreasing the visual quality towards the periphery, foveated rendering ensures the efficient use of valuable computational resources in real-time, without comprising visual quality perceived by users.

The tight spot in rendering

Computer graphics is the field of generating images and videos using computers. To create an image, a computer usually requires a geometric and visual description of the scene together with objects and light sources inside the scene as input. Then, the computer generates an image from the given description of the scene as seen by a camera that is virtually placed inside the scene. This process resembles taking a photo of a scene with a camera in the real world.

When we take a photo in real life, the camera provides us with the image it "sees" when the light rays from the scene hit the sensor or photographic film. This image results from physical interactions between the light and the scene, which defines the final color and brightness of the light when it reaches the camera. When a light ray hits a surface, it gets reflected, absorbed, or transmitted (as we see in transparent materials like glass), as illustrated in Figure 4. Furthermore, the properties of object surfaces in the scene, their position, and angle with respect to the direction of the incident light, and the camera position also play a role in the interaction.

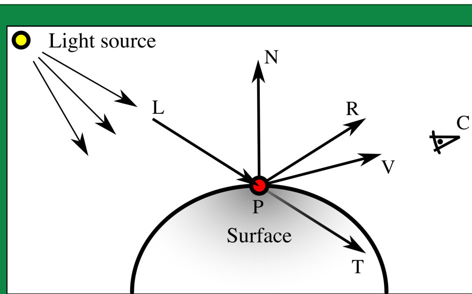


Figure 4: Incoming light (L) from a light source can get absorbed, reflected (R), or transmitted (T) when interacting with a surface point (P) before it reaches the camera/eye (C).
The normal vector (N) perpendicular to the surface at P and the vector (V) to the camera/eye can be used to estimate the resulting color observed at P.

A computer's job in generating images and videos is not easy because it has to simulate those interactions between light and scene virtually, in a process called *rendering*. Obtaining a realistic, high-quality image via rendering requires a very accurate and expensive computation of complex interactions between the light and the scene. Furthermore, our displays keep improving in ways that increase the computational cost of rendering. As resolution increases, newer displays are able to showcase finer details, putting pressure on the renderer to keep up. The use of *Virtual reality* (VR) displays is on the rise, and this necessitates rendering the scene twice, for the left and the right eye. The use of *Augmented reality* (AR) headsets involves resource-intensive tasks, such as aligning virtual objects with real-world geometry, which is an extra computational expense. With these new developments, the task of maintaining the desired rendering quality has become even more demanding due to the increased computational requirements from our limited hardware resources.

When it comes to creating top-notch images and videos using computers, our focus is typically on utilizing advanced graphics processing units (GPUs) that are specifically designed for handling complex graphics tasks. However, this comes with challenges. First, the capabilities of GPUs are typically reflected in their corresponding purchase prices, and under a limited financial budget, we may have to explore alternative options instead of obtaining a top-of-the-line GPU. Even with no financial constraints, deploying new hardware into existing systems may be hindered by limited physical access. Secondly, as the accuracy and complexity of rendering calculations improve, so does the energy consumption, a significant concern for battery-powered mobile devices. Finally, to accurately generate high-quality and realistic images, a greater amount of visual data must be stored, resulting in a larger storage space requirement. Due to technical constraints in transmitting and storing computer-generated images, it may not be possible to retain all necessary image details for optimal visual quality.

Our ultimate goal in computer graphics is to maintain a satisfactory level of visual quality. But, while pursuing this goal, can we simultaneously conserve resources, enhance performance, and achieve energy efficiency?

Render wars – A new hope

As computer graphics developers and researchers grapple with the balance between achieving high visual quality and managing rendering costs, they have begun exploring solutions to maximize the efficiency of limited computational resources. Recognizing that humans are the primary viewers of computer-generated images and videos, a notable strategy has emerged from the field of *visual perception*, which focuses on understanding the characteristics of the human visual system (HVS).

Visual perception provides valuable insights for optimizing graphics to match the capabilities of human vision. The process of human visual processing begins in the eyes, where light interacts with photoreceptors on the retina. These photoreceptors are distributed unevenly across the retina. The *fovea*, a region spanning about 5 degrees in the center of our visual field, is densely packed with photoreceptors that are particularly sensitive to color and intricate details. In contrast, our peripheral vision is characterized by photoreceptors that are sparser and more attuned to low light and detecting motion (Figure 5). Consequently, our ability to perceive fine details and colors diminishes as an image moves from the fovea to the periphery of our visual field. To experience this phenomenon first-hand, try a simple experiment: focus on a word in this paragraph and then attempt to read the words above it without moving your eyes. While this may be a bit challenging, you will find that you can recognize words at most 2 or 3 lines away using your peripheral vision, illustrating the variations in visual acuity at different parts of the retina.

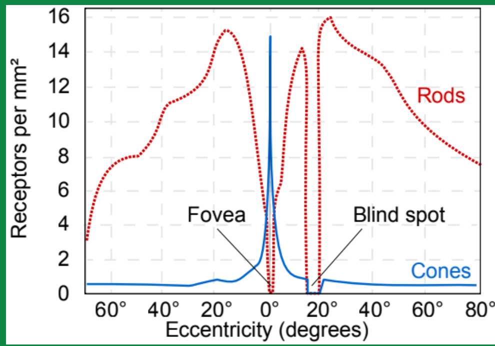


Figure 5: The distribution density of the two types of photoreceptors in the human retina; **cones** and **rods**. The horizontal axis indicates the angular distance from the fovea, the center of our vision. **Cones**, which are active in well-lit conditions and enable color vision, are concentrated around fovea. **Rods**, which facilitate night vision are more dense in the peripheral areas of the retina. This variance in the density and distribution of photoreceptors underpins the difference in visual capabilities between central and peripheral vision [Wan95].

In the realm of rendering, there is a constant pursuit for the highest possible resolution and quality in images and videos, often achieved by densely packing as many pixels as possible into our displays and cameras. However, unlike the human retina, where photoreceptors are unevenly distributed, the pixels in displays or camera sensors are arranged uniformly as a rectangular grid. Given this, devoting equal computational resources to render every part of an image or video in the highest

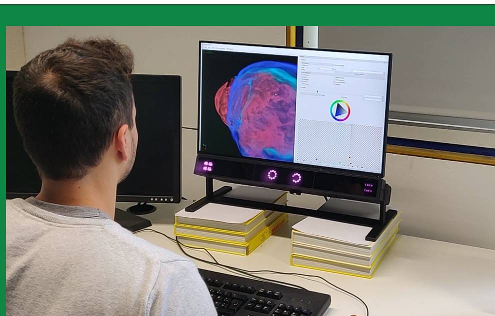


Figure 6: A foveated rendering system consisting of a desktop display and an eye tracker positioned beneath the display.

quality seems inefficient, and a more resource-efficient approach involves the use of an *eye tracker* to determine where a viewer is looking on a display (Figure 6). By monitoring a viewer's gaze position on the display, it becomes possible to strategically allocate rendering resources by reducing the quality in the peripheral vision where finer details are less discernible. This method ensures high-quality rendering where it matters most while conserving computational resources in areas less critical to the viewer's immediate focus. Inspired by the human retina, this technique is called *foveated rendering*.

Types of foveated rendering

By reducing the rendering quality in the peripheral vision, foveated rendering significantly reduces the computational load on the system. This allows for higher performance or it can enable more complex scenes or higher resolutions in the foveal region than would otherwise be possible. Many different flavors of foveated rendering have been developed over time, each employing different techniques to optimize the graphical performance. We describe several examples of this in the following paragraphs.

Standard dynamic foveated rendering

Foveated rendering, in its simplest form, organizes the visual field into concentric zones around the point of gaze, progressively decreasing rendering quality (such as resolution) with increasing distance from the fovea [GFD+12] This approach leverages a human perception model informed by the distribution of photoreceptor cells (Figure 5), making it relatively straightforward to implement. Consequently, as one moves further from the viewer's point of gaze, the rendering quality on the display diminishes. This concept is illustrated in the rendering resolution map for the standard dynamic foveated rendering technique, depicted in Figure 7 (b). Techniques such as contrast enhancement in post-processing or the introduction of synthesized noise, which replicates the spatial details lost due to reduced rendering quality, can help mitigate the visibility of this quality reduction [PSK+16, TTD22].

Content-aware foveated rendering

The standard dynamic foveated rendering maintains a constant quality across different visual contents, disregarding the nature of the content itself. This approach can lead to varying levels of visibility in quality reduction, especially in areas with complex details or high contrast. On the other hand, content-aware foveated rendering adapts the quality based on both the viewer's gaze distance and the complexity of the visual content [TAKW+19]. Implementing this adaptive approach is more complex because it requires adjusting the rendering quality based on two criteria: the proximity to the gaze point and the intricacy of the content. Additionally, these adjustments must be made using a simplified version of the scene to ensure efficiency. Rendering the scene in full resolution for quality assessment would negate the benefits of reduced rendering, thus the challenge lies in making these decisions based on a representative, lower-resolution preview of the scene. A sample resolution map for this technique is shown in Figure 7 (b), corresponding to the 3D scene in Figure 7 (a).

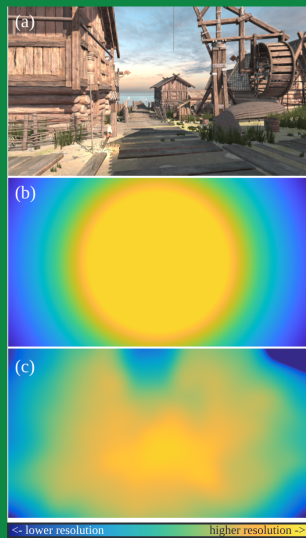


Figure 7: For a 3D input scene (a), images (b) and (c) show the color-coded rendering resolutions achieved through the standard dynamic foveated rendering and the content-aware foveated rendering, respectively. In both cases, it is assumed that the viewer is looking at the center of the display. Content-aware foveated rendering optimizes the use of the rendering budget by considering the visual content when determining the resolution.

Foveated tessellation

Objects and characters in digital scenes, such as for computer games or animated movies, are typically described using meshes, i.e., collections of polygons such as triangles. While GPUs are increasingly more capable of quickly processing and rendering models with lots of triangles, transferring these from the CPU to the GPU every frame in real-time applications quickly becomes a bottleneck for animated models (i.e., models where the positions of the vertices/triangles keep changing). To alleviate this, a coarse (or even compressed) description of the model is sent from the CPU, and the details are then interpolated using various smooth representation methods, such as splines or subdivision (the latter being the animation's industry standard), using a technique called tessellation on the GPU. However, when used naively, it leads to (unnecessarily) dense models that need to be rendered. Enter foveated tessellation. Based on the user's gaze/center of the field of view, we can use tessellation adaptively, inventing more triangles for the foveal area while leaving the peripheral region comparably sparse [TRK20]. The example in Figure 8 shows a five-fold reduction in the number of triangles needed to render a smooth-appearing model with foveated tessellation compared to standard (naive) model tessellation.

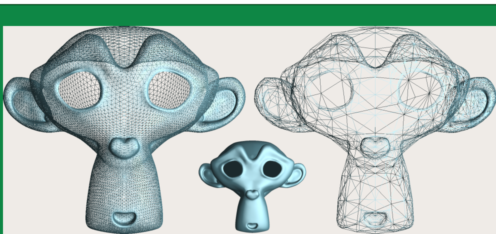


Figure 8: Foveated tessellation is demonstrated on a monkey head model (known as Blender's Suzanne), which consists of 856 curved faces (middle image). The foveated approach utilizes only 20% of the triangles compared to the fully tessellated model. On the left, the model is globally tessellated, comprising 50k triangles. On the right, foveated tessellation is applied, resulting in 10k triangles. This technique adaptively generates triangles on the GPU, focusing only on the regions the (virtual) camera or user is observing – in this case, the right ear.

Foveated volume ray-casting

Volume ray-casting is a technique used for rendering 3D volumetric data. When adapting this technique for foveated rendering, a unique approach is necessary due to the inherent characteristics of volume ray-casting. One effective strategy integrates the gradual decrease in human visual acuity towards the periphery with specific sampling and interpolation methods suitable for volumetric data, such as Linde-Buzo-Gray sampling and natural neighbor interpolation as illustrated in Figure 9 [BSB+19]. This approach involves rendering samples of 3D volumetric data, as depicted in Figure 9(a), which are then used as input for the reconstruction process shown in Figure 9(b). The interpolation technique employed for reconstruction is more computationally efficient than densely sampling the volumetric data, as is done in full-resolution rendering. This leads to significant enhancements in rendering performance with a minimal impact on visual quality.

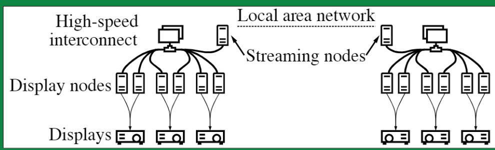
(a) Image space sampling

(b) Reconstruction

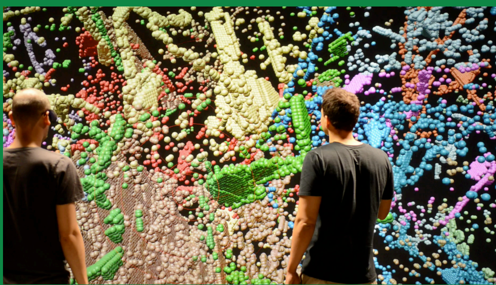
Figure 9: Voronoi-based foveated volume ray-casting result [BSB+19]

Foveated streaming

Foveated rendering approaches can further facilitate collaborative exploration of scientific data sets across large high-resolution displays, which requires both high visual detail as well as low-latency transfer of image data (Figure 10).



(a) Overview of the technique



(b) An application on a large display, known as *powerwall*

Figure 10: Foveated streaming for large high-resolution displays [FBB+21]. The diagram in (a) shows an overview of the method, whereas the image in (b) shows the method in action with multiple viewers observing an application with particle rendering.

Using foveated rendering to dynamically adapt the encoding quality while streaming the data reduces the required bandwidth [FBB+21] (a). Different from single-user foveated rendering, this technique requires tracking the gaze of multiple viewers, and respectively adapting the quality parameter of the video encoder. This allows us to substantially reduce the required bandwidth for transferring the data with low latency, which is crucial for collaborative analysis across display walls at different physical locations. An overview and a sample use case of this technique is depicted in Figure 10.

Conclusion

We have described foveated rendering and its successes in a selected set of different applications. But the story does not end here. With the popularity of the Augmented and Virtual Reality on the rise, methods exploiting foveated methods will attain more widespread use with more challenging use cases that require creative solutions.

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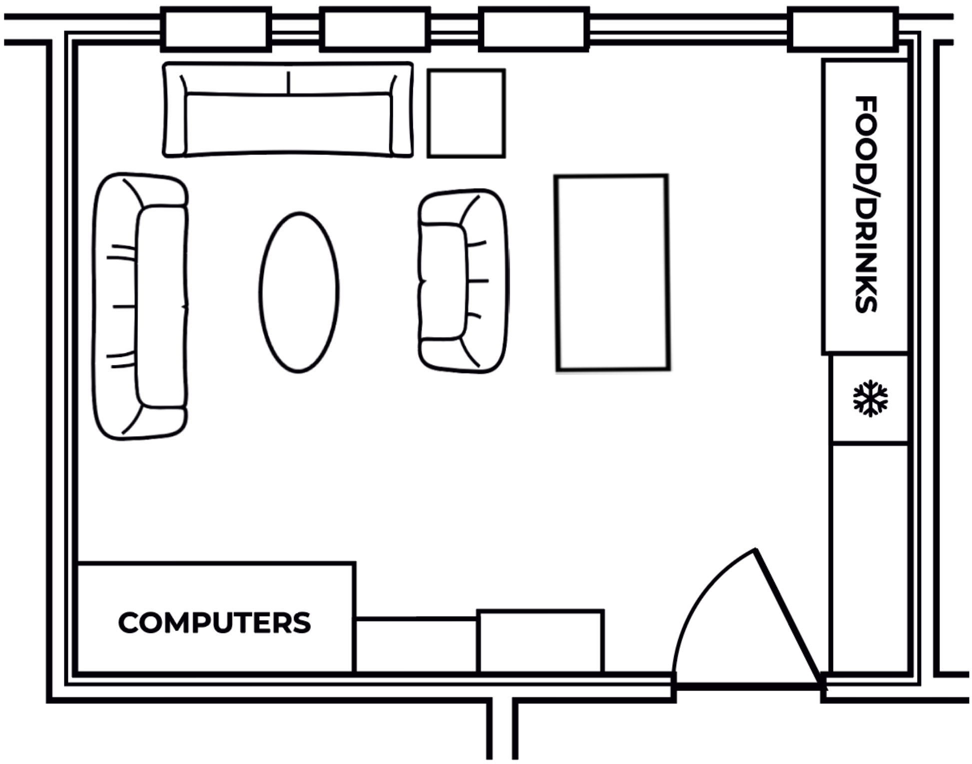
Arrange your own FMF room

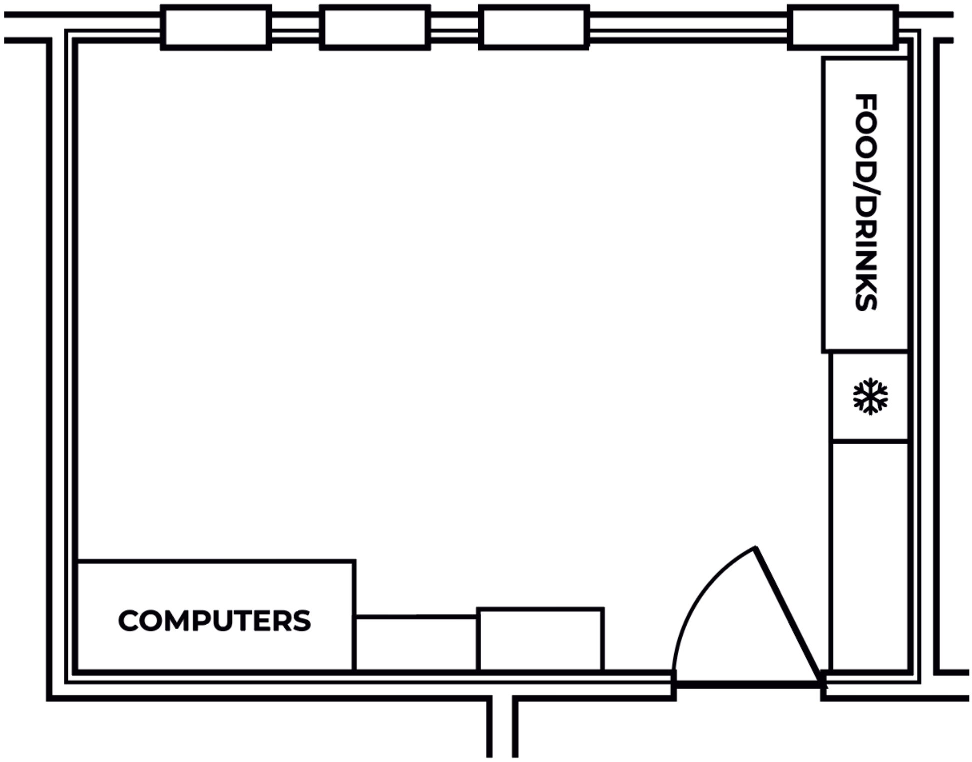
AUTHOR: **R. MOL**

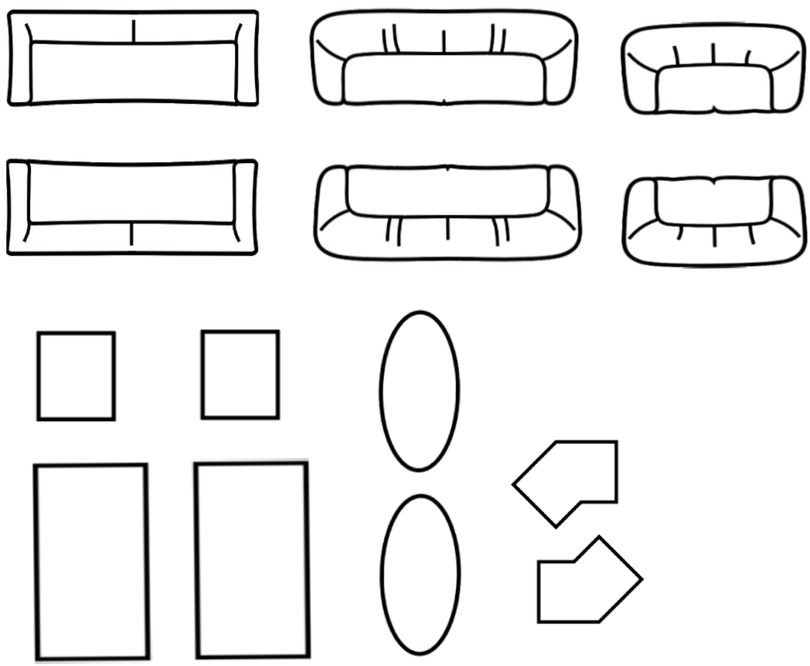


Room 5111.0053 has been the FMF room since 2008. Affectionally dubbed the New Super FMF Workroom (NSFW), the FMF room has been the home of the FMF and its members for as long as any of us can remember.

With the FMF moving to the Feringa Building, we thought it would be nice to reminisce and rearrange the FMF one last time.

On the next page, cut out and move the furniture to your liking onto the empty version of room 5111.0053. Send us your best arrangements at perio@fmf.nl!


FOOD/DRINKS
COMPUTERS



Recipe
Mizeria

AUTHOR: **B. FIBAKIEWICZ**

For the recipe of this edition we have Mizeria, a refreshing Polish cucumber salad. Its name derives from the word zmizerować: to wane. It is cheap and easy to make, and works excellently as a soft meal for breakfast or a side dish for dinner.

Ingredients

- 500 grams/2 fresh cucumbers
- 5 spoons of smetana (sour heavy cream)
- 2-3 teaspoons of salt
- A sprinkle of dill

Optionally:

- 2 teaspoons of lemon juice
- Soy sauce

Instructions

Peel the skin off the cucumbers and then, using the peeler, cut it into thin vertical slices. Cover the slices with salt and let it sit for at least thirty minutes (preferably in the fridge) i.e. until the water has thoroughly left the cucumber, from which the name is derived. Afterwards add the smetana and sprinkle in the dill.

NB: Smetana is a cream with over 30% fat: one way of acquiring it is through the Polish shop in Groningen, otherwise consider using any higher fat non curdling cream.

Figure 11: Mizeria

Brainwork

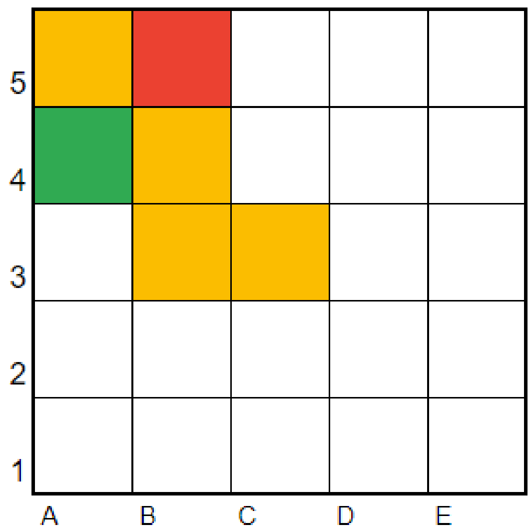
Colour-full

AUTHOR: **R. MOL**

Fill the grid below by colouring each square either blue, green, red, white, or yellow and using the hints below. 6 squares are given as a start. Each set of 3 hints contains 1 false hint and two true hints. In this context, “neighbouring” squares are only squares to the left, right, above, or below unless specified otherwise.

Some hints contain an addition bit of information about a specific square indicated in square brackets. If one of these is true, then both are true and if one of these is false, then both are false.

Mail your solution to Perio@fmf.nl!



1. There exists exactly 1 column that contains all colours.
2. There exists a colour, other than yellow, such that each yellow square neighbours this colour.
3. There exists a white square that neighbours only red squares.

4. Each blue square neighbours exactly 1 other blue square. **[C2 is Green]**
5. Each colour appears equally often. **[B1 is Blue]**
6. No red square neighbours a red square or is diagonally connected to a red square and no green square neighbours a green square. **[D4 is Blue]**

7. All rows and columns are unique.
8. At least one of the rows contains the Romanian flag (from left to right: Blue, Yellow, Red).
9. Each column contains at least one red square.

10. There exists exactly 1 white square that neighbours every other colour.
11. One of the diagonals contains all colours. **[E4 is Blue]**
12. If a 2x2 area has 3 squares of the same colour, then all squares in that 2x2 area have the same colour.

13. The middle row has exactly two colours, each appearing at least twice. **[D5 is Green]**
14. A1 has the same colour as A2.
15. There are exactly 2 rows with exactly 3 green squares each. **[C5 is white]**

Solution to the previous Brainwork

The gravitational force between two objects is given by

$$F = G \cdot \frac{m_1 m_2}{r^2},$$

with G a constant, m_1 and m_2 the masses of the objects in kilograms and r the distance between them in kilometers.

Mass of the ISS[3]:	419,725 kg
Mass of the moon[4]:	$7.346 \cdot 10^{22}$ kg
Average Mass of a person[5][6]:	62 kg
Number of people on Earth[7]:	$8.1 \cdot 10^9$
Distance from the moon to the Earth[8]:	384,400 km
Distance from the Earth to the ISS[9]:	400 km
Diameter of Earth[10]:	12,756 km

This makes the forces equal to

$$F_{Astroanut,Moon} = G \cdot \frac{80 \cdot 7.346 \cdot 10^{22}}{(384,400 - 400)^2} = 3.9855 \cdot 10^{13} \cdot G.$$

$$F_{People,ISS} = G \cdot \frac{62 \cdot 8.1 \cdot 10^9 \cdot 419,725}{(400 + 6378)^2} = 4.588 \cdot 10^9 \cdot G$$

Meaning that the gravitational force between the austronaut and the moon is 10,000 times as large as the gravitational force between all people on Earth and the ISS.

[3] https://www.nasa.gov/international-space-station/space-station-facts-and-figures/
[4] https://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html
[5] https://www.syfy.com/syfy-wire/the-human-cube-the-volume-of-humanity
[6] We estimate an astronaut is slightly heavier at 80kg
[7] https://www.worldometers.info/world-population
[8] https://www.rmg.co.uk/stories/topics/how-far-away-moon
[9] https://www.jpl.nasa.gov/edu/teach/activity/how-far-away-is-space
[10] https://coolcosmos.ipac.caltech.edu/ask/57-How-large-is-Earth